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# The method of IIP-weighted indices of exchange rate effects using quarterly chain linking[1]

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**Abstract**:

A notable percentage of financial assets and liabilities in Germany's international investment position (IIP) are denominated in a foreign currency. Exchange rate movements therefore have a major impact on trends in the IIP. A newly developed index concept now allows more in-depth analyses. The indices of the exchange rate effects in the international investment position (IIE) show how the value of external assets changes solely as a result of exchange rate movements. IIE are highly granular and allow conclusions to be drawn about the impact of changes in the prices of individual currencies on asset and liability holdings broken down by sector and instrument.

The IIE allow methodologically advanced risk analyses. Also, sensitivity analyses can be used to identify individual sectors which would, in certain scenarios, be majorly affected by (assumed) changes in the prices of individual currencies. In addition, methods for analysing time series can be applied to the indices, for example to measure the exchange rate-related volatility of the market value of individual asset holdings.

When interpreting the IIE as a measure of risk for changes in the value of assets in individual domestic sectors as a result of exchange rate movements, it should be noted, however, that the hedging operations which financial market players use to reduce their currency risk are not taken into consideration. Moreover, no account is taken of individual enterprises' option of offsetting their exchange rate risk within an international group.

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## Introduction[3]

The newly developed indices of the exchange rate effects in the international investment position (IIE) of Germany show how the value of external assets changes solely as a result of exchange rate movements.

The concept is based on a system of weighted exchange rates. The index system weights are based on IIP stocks broken down by currency, sector and asset class and listed separately for assets and liabilities. The weighting of the IIE is based on the most recent data available, as abrupt transaction-related adjustments of asset or liability positions in the IIP cannot be ruled out.[4]

On the basis of the quarterly IIP data, which has been available broken down by currency since the end of 2012, quarterly Laspeyres-type chain links are constructed (section 1.1). In order to obtain multi-period indices for exchange rates, the quarterly links are continuously chain-linked (section 1.2). The continuous chain-linked IIE have similar characteristics to the better-known chain indices of the annual-overlap or monthly-overlap type, which are used, for instance, to calculate price-adjusted gross domestic product or harmonised consumer prices (section 2).

Because of the specific character of the continuous chain-linked indices, a deeper understanding of their properties is developed. Finally programmes are produced for the IIE that enable aggregation and disaggregation of indices as well as the calculation of growth contributions to the exchange rate-related percentage change in an aggregate (section 3).

# 1 Derivation of IIP-weighted exchange rates

### 1.1 Laspeyres-type chain links

On the basis of the international investment position data that has been available, broken down by currency, since 2013, non-transaction-related changes, i.e. market price effects, exchange rate effects and other adjustments, can now also be calculated. Exchange rate effects[5] are calculated as:

(1) $$ERE_t^{i,s} = \sum_k E_t^k A_{t-1}^{k,i,s} - \sum_k E_{t-1}^k A_{t-1}^{k,i,s}$$

with

[3] See Deutsche Bundesbank (2018), pp 36-37.
[4] Contrary to former own calculations of IIP-weighed exchange rates and the work done by Lane & Shambough (2010), this work uses quarterly instead of annual data. Additionally, the granular data available allows a richer differentiation in respect to sector, asset and currency classes.
[5] The exchange rate effects set forth in the IIP report take into account not only the effects of exchange rate movements on asset holdings but also on the transactions carried out in each respective quarter. The exchange rate effects of transactions are generally of secondary importance.

$ERE_t^{i,s}$ the exchange rate effect for asset class $i$ ($i = 1, \dots, N$) in domestic sector $s$ ($s = 1, \dots, S$) in period $t$ ($t = 1, \dots, T$) in €.

$E_{t-1}^{k}$ the exchange rate of currency $k$ ($k = 1, \dots, K$) at the end of period $t - 1$ (or at the beginning of period $t$). Exchange rates are cited using the direct quotation method (e.g. US$1 = €0.87) There is no exchange rate conversion for any euro-denominated assets, i.e. $E = 1$.

$A_{t-1}^{k,i,s}$ the holdings of asset class $i$ mapped to sector $s$, denominated in currency $k$ at the end of period $t - 1$.

With the introduction of the euro, the ECB's euro reference exchange rates for the 30 most important currencies are cited using the indirect quotation method (e.g. €1 = US$1.15). As a result, indirect quotations are available especially for the currencies that are explicitly included in the calculation of the IIP-weighted exchange rate. The available exchange rates cited using the indirect quotation method ($EM$) are converted to the desired direct quotation ($E$) through inversion:

(2) $$E_t = {}^{1}/_{EM_t}$$

The exchange rate effects (ERE) on the previous period's stocks of external assets published in the quarterly IIP report are used to calculate growth factors. These growth factors are also the chain links (IE) which are later used to derive chained Laspeyres indices of IIP-weighted exchange rate effects (IIE).

(3) $$IE_t^{i,s} = \frac{ERE_t^{i,s}}{\sum_k E_{t-1}^{k} A_{t-1}^{k,i,s}} + 1$$

with

$IE_t^{i,s}$ the growth factor for the exchange rate effect on asset class $i$ of sector $s$ at the end of period $t$.

If the calculation instruction used in the IIP report (1) is plugged into (3) for the exchange rate effect ERE, this yields the following:

(4) $$IE_t^{i,s} = \frac{\sum_k E_t^{k} A_{t-1}^{k,i,s} - \sum_k E_{t-1}^{k} A_{t-1}^{k,i,s}}{\sum_k E_{t-1}^{k} A_{t-1}^{k,i,s}} + 1 = \frac{\sum_k E_t^{k} A_{t-1}^{k,i,s}}{\sum_k E_{t-1}^{k} A_{t-1}^{k,i,s}}$$

This results in a weighted exchange rate effect in line with Laspeyres for asset class $i$ and sector $s$. Aggregated across all sectors and asset classes, the IIP-weighted (overall) exchange rate effect is as follows.

(5) $$IE_t = \frac{\sum_k \sum_i \sum_s E_t^{k} A_{t-1}^{k,i,s}}{\sum_k \sum_i \sum_s E_{t-1}^{k} A_{t-1}^{k,i,s}}$$

The growth factor shows the exchange rate-related change of the total assets or liabilities in Germany's international investment position in period $t$ compared to the base period. It states the relative change in assets directly.

Equation (5) can also be used as a starting point for sensitivity analyses or forecasts. In particular, if its value is calculated using an exchange rate forecast for a currency $k$ at time $t$ and unchanged rates for the remaining currencies at the previous period's level (*ceteris paribus*), the result shows the effects of the exchange rate change of this one currency on the IIP.

The calculation of totals for the IE across the three observation levels (currency, asset class and sector) also provides the possibility of an isolated observation of the exchange rate effect for each individual dimension. As a result, growth factors of the type as in (5) for sector $s$

(6) $$IE_t^s = \frac{\sum_k \sum_i E_t^k A_{t-1}^{k,i,s}}{\sum_k \sum_i E_{t-1}^k A_{t-1}^{k,i,s}} \qquad \text{for } s \in [1;S]$$

or asset class $i$

(7) $$IE_t^i = \frac{\sum_k \sum_s E_t^k A_{t-1}^{k,i,s}}{\sum_k \sum_s E_{t-1}^k A_{t-1}^{k,i,s}} \qquad \text{for } i \in [1;N]$$

make it possible to make statements on the exchange rate effects in each defined segment.[6] They are interpreted in a manner similar to the overall index.

### 1.2 Calculating multi-period indices of the Laspeyres IIP-weighted exchange rates

The international investment position is reported on a quarterly basis. To this end, the above-mentioned equation (4) and (5) are used to generate growth factors which document the exchange rate-related change in foreign assets compared with the preceding quarter. In formal terms, growth factor for quarter $Q$ in year $J$ is as follows:

(8) $$IE_{J,Q}^i = \frac{\sum_i E_{J,Q}^i A_{J,Q-1}^i}{\sum_i E_{J,Q-1}^i A_{J,Q-1}^i}$$

To simplify notation, the superscript elements $s, k$ and $i$ were aggregated to deliver a single running index $i$ for each currency-sector-asset class combination. Another convention is that the nulled quarter of year $J$ is automatically the last quarter of the preceding year, that is to say $J,0 = J-1,4$. This ensures that the growth factor in the first quarter $IE_{J,1}^i = \frac{\sum_i E_{J,1}^i A_{J,0}^i}{\sum_i E_{J,0}^i A_{J,0}^i} = \frac{\sum_i E_{J,1}^i A_{J-1,4}^i}{\sum_i E_{J-1,4}^i A_{J-1,4}^i}$ is likewise captured by formula (8).

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[6] A selection of a single currency $k$ in (5) does not produce any new findings because this sub-index matches the change in the exchange rate under consideration.

Simple remodelling of (8) also provides a standard notation format for the growth factors that highlights changes in the weighted exchange rates.

(9) $$IE^i_{J,Q} = \sum_i \frac{E^i_{J,Q}}{E^i_{J,Q-1}} g^i_{J,Q-1} \qquad \text{where } g^i_{J,Q-1} = \frac{E^i_{J,Q-1} A^i_{J,Q-1}}{\sum_i E^i_{J,Q-1} A^i_{J,Q-1}}$$

Germany's euro-denominated external assets, standing for the sector asset-class combination $i$ in relation to the entire IIP in the preceding quarter, are calculated directly on the basis of the reported IIP and serve as weighting shares $g^i_{J,Q-1}$. These indicate the relative significance of exchange rate movements for the currency used in combination $i$. Because of their immediacy, the cumulative exchange rate movement is very up-to-date and is not affected by an obsolete weighting structure.

The long-term evolution of an exchange rate effect index can be arrived at using the continuous multiplication (chain-linking) of all quarterly growth factors.

(10) $$IIE^i_{J,Q} = 100 \cdot IE^i_{1,1} \cdot IE^i_{1,2} \cdot \ldots \cdot IE^i_{J,Q} = IIE^i_{J,Q-1} \cdot IE^i_{J,Q}$$

$IIE^i_{J,Q}$ refers to an index in year $J$ and quarter $Q$, chained across several periods. The term after the second equals sign indicates that a current chain index value is created by multiplying the previous chain index value by the current growth factor. The fourth quarter in year 0 is the reference period in which a chain index value of 100 is implicitly given.

## 2 Characteristics of the IIE chain indices

### 2.1 Effects of the ongoing updating of weights

Exchange rate movements as well as volume effects can be depicted in a single index by updating the weights on an ongoing basis.

The weights applied for short-term exchange rate movements (see (9)) are always taken from the preceding period. Quarterly chain-linking is necessary on account of the quarterly change in the basis and in the weights relating to the growth factors. However, over time such chain-linking results in the evolution of the index being influenced not just by exchange rate movements but also by changes in the external asset holdings contained in the weights. This needs to be taken into consideration when interpreting the chain indices.

The multiplicative linking of the growth factors can cause the chain index values of two non-consecutive points in time characterised by identical exchange rates and IIP values to differ. This defining feature of all chain indices is called path dependency.[7]

[7] See von der Lippe (2001), p 30.

Official statistics handbooks generally advise against chain-linking Laspeyres indices in the course of a given year[8] as, given the cyclical fluctuations in quantities and prices that will arise, this carries the risk of divergence (also known as drift) compared with a less frequently chained index.[9] Since greater importance is attached to the benefits of timely weighting in this particular case, the drift aspect has to be disregarded.

### 2.1.1 Period-on-period comparisons and growth contributions

Calculating the change in the chain index relative to its value in the immediately preceding period directly produces the growth factor used for the purpose of chain-linking:

(11) $$\frac{IIE_{J,Q}^{i}}{IIE_{J,Q-1}^{i}} = IE_{J,Q}^{i}$$

The growth factors, on the other hand, use identical quantities in the numerator and denominator, and therefore solely depict exchange rate movements.

The following procedure is applied to calculate the quarter-on-quarter IIE percentage changes:

(12) $$\frac{IIE_{J,Q}^{i}}{IIE_{J,Q-1}^{i}} - 1 = IE_{J,Q}^{i} - 1$$

The Laspeyres growth factors on which the IIE are based are aggregatively consistent and can be weighted to the overall growth. In the same way the quarter-on-quarter percentage change rates of an aggregate can be summed up from the growth contributions of its components. Using the aggregated growth factor

(13) $$IE_{J,Q}^{A} = \frac{\sum_a E_{J,Q}^{a} A_{J,Q-1}^{a}}{\sum_a E_{J,Q-1}^{a} A_{J,Q-1}^{a}} \qquad a = 1, \dots, L, M, \dots, Z$$

and the growth factors of two sub-indices

(14) $$IE_{J,Q}^{L} = \frac{\sum_l E_{J,Q}^{l} A_{J,Q-1}^{l}}{\sum_l E_{J,Q-1}^{l} A_{J,Q-1}^{l}} \qquad l = 1, \dots, L$$

(15) $$IE_{J,Q}^{M} = \frac{\sum_m E_{J,Q}^{m} A_{J,Q-1}^{m}}{\sum_m E_{J,Q-1}^{m} A_{J,Q-1}^{m}} \qquad m = M, \dots, Z$$

it is possible to show the percentage change in the aggregate as the weighted sum of the percentage changes in the components.

(16) $$IE_{J,Q}^{A} - 1 = \left(IE_{J,Q}^{L} - 1\right) g_{J,Q-1}^{L} + \left(IE_{J,Q}^{M} - 1\right) g_{J,Q-1}^{M}$$

The weighting shares correspond to Germany's external assets (converted into euro) of the sub-components as a share of the aggregate's IIP

(17) $$g_{J,Q-1}^{L} = \frac{\sum_l E_{J,Q-1}^{l} A_{J,Q-1}^{l}}{\sum_a E_{J,Q-1}^{a} A_{J,Q-1}^{a}}$$ while

[8] See International Labour Organization (2004), section 15.84, and European Commission et al (2008), sections 15.43 et seq., and Eurostat (2013), section 6.32.
[9] See Forsyth & Fowler (1981): 236, 240 et seq., and European Commission et al. (1993), sections 16.44-16.49.

(18) $$g^{M}_{J,Q-1} = \frac{\sum_m E^{m}_{J,Q-1} A^{m}_{J,Q-1}}{\sum_a E^{a}_{J,Q-1} A^{a}_{J,Q-1}}$$

The growth contributions of components $L$ $\left(CTG^{L}_{J,Q|1}\right)$ and, accordingly, $M$ $\left(CTG^{M}_{J,Q|1}\right)$ to the quarterly percentage change in its aggregate are therefore shown as weighted percentage growth rates of the respective component, with the subscript "1" used in the growth contribution $(CTG)$ to denote the quarter-on-quarter change:

(19) $$CTG^{L}_{J,Q|1} = (IE^{L}_{J,Q} - 1) g^{L}_{J,Q-1}$$

Equation (16) ensures that the sum of the growth contributions of components $L$ and $M$ is equal to the percentage change in the aggregate.

### 2.1.2 Year-on-year comparisons and growth contributions

Comparisons across several quarters and, in particular, with the corresponding quarter in the previous year, present a distorted picture. This consequently also applies to the calculation of annual changes at year-end. The change relative to the corresponding quarter in the previous year is calculated as

(20) $$\frac{IIE^{i}_{J,Q}}{IIE^{i}_{J-1,Q}} = \frac{100 \cdot IE^{i}_{1,1} \cdot IE^{i}_{1,2} \cdot \ldots \cdot IE^{i}_{J,Q}}{100 \cdot IE^{i}_{1,1} \cdot IE^{i}_{1,2} \cdot \ldots \cdot IE^{i}_{J-1,Q}} = IE^{i}_{J,Q-3} \cdot \ldots \cdot IE^{i}_{J,Q}$$

and is itself once again a chained representation. All quarters smaller than 1 – in line with the convention introduced in connection with (8) – fall within the previous year. Since different weights are used for each of the quarterly growth factors in (20), changes in quantity are also reflected in changes between two non-consecutive periods. Pure comparisons of the exchange rate movements are not possible.

The percentage change relative to the corresponding quarter in the previous year is:

(21) $$\frac{IIE^{i}_{J,Q}}{IIE^{i}_{J-1,Q}} - 1 = IE^{i}_{J,Q-3} \cdot \ldots \cdot IE^{i}_{J,Q} - 1$$

The year-on-year percentage change of a continuously chained index can be shown as a sum. In this context, each summand corresponds to the product of a quarter-on-quarter percentage change and reflects past growth factors relating to the sub-chain in (20). Thanks to the aggregative consistency of these year-on-year percentage changes, they can in turn be presented as weighted sums of the components. The growth rate of a quarterly chained IIE aggregate is calculated as follows:

(22) $$\begin{aligned}\frac{IIE^{A}_{J,Q}}{IIE^{A}_{J-1,Q}} - 1 = \quad & \left[\left(IE^{L}_{J,Q-3} - 1\right) g^{L}_{J,Q-4} + \left(IE^{M}_{J,Q-3} - 1\right) g^{M}_{J,Q-4}\right] + \\ & IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q-2} - 1\right) g^{L}_{J,Q-3} + \left(IE^{M}_{J,Q-2} - 1\right) g^{M}_{J,Q-3}\right] + \\ & IE^{A}_{J,Q-2} \cdot IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q-1} - 1\right) g^{L}_{J,Q-2} + \left(IE^{M}_{J,Q-1} - 1\right) g^{M}_{J,Q-2}\right] + \\ & IE^{A}_{J,Q-1} \cdot IE^{A}_{J,Q-2} \cdot IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q} - 1\right) g^{L}_{J,Q-1} + \left(IE^{M}_{J,Q} - 1\right) g^{M}_{J,Q-1}\right]\end{aligned}$$

The growth factors of the aggregate and of the components, as well as the weights, are defined in accordance with (13) to (18).

The growth contribution of component L $\left(CTG^{L}_{J,Q|4}\right)$ and, accordingly, M $\left(CTG^{M}_{J,Q|4}\right)$ to the percentage change in the aggregate compared with the corresponding quarter in the previous year (denoted by the subscript |4) is therefore given by

$$(23)\qquad CTG^{L}_{J,Q|4} = \left[\left(IE^{L}_{J,Q-3}-1\right)g^{L}_{J,Q-4}\right] + IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q-2}-1\right)g^{L}_{J,Q-3}\right] + IE^{A}_{J,Q-2}\cdot IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q-1}-1\right)g^{L}_{J,Q-2}\right] + IE^{A}_{J,Q-1}\cdot IE^{A}_{J,Q-2}\cdot IE^{A}_{J,Q-3}\left[\left(IE^{L}_{J,Q}-1\right)g^{L}_{J,Q-1}\right]$$

Here, too, the sum of the growth contributions of the components delivers the overall percentage change. The basic idea of the growth contributions calculated in this way corresponds to the principle that Ribe[10] uses as the basis for calculating growth contributions to the annual change rate in other chain-linking concepts.

### 2.1.3 Consistency in changes between IIP and IIE

As was seen in section 1, the reported exchange rate effect ERE served as the basis for calculating the index. Therefore, ERE on the previous period's stocks of external assets, calculated by (3), is consistent with the quarter-on-quarter changes in the IIE, measured as growth factor by (11).

Contrary, statements on annual developments between the IIE and the exchange rate effects reported in the IIP differ. For example, the annual growth of a given IIE is arrived at using the product of the intra-year growth factors.

$$(24)\qquad IE^{i}_{J} = \prod_{Q=1}^{4} IE^{i}_{J,Q} = \frac{\sum_i E^{i}_{J,1}A^{i}_{J,0}}{\sum_i E^{i}_{J,0}A^{i}_{J,0}}\cdot\frac{\sum_i E^{i}_{J,2}A^{i}_{J,1}}{\sum_i E^{i}_{J,1}A^{i}_{J,1}}\cdot\frac{\sum_i E^{i}_{J,3}A^{i}_{J,2}}{\sum_i E^{i}_{J,2}A^{i}_{J,2}}\cdot\frac{\sum_i E^{i}_{J,4}A^{i}_{J,3}}{\sum_i E^{i}_{J,3}A^{i}_{J,3}}$$

By contrast, the annual exchange rate effects reported in the IIP are calculated as the sum of the quarterly results. Where this aggregate is, say, based on the IIP stocks at the end of the previous year, this produces a different development, as demonstrated in (24).

## 3 KIX plug-in for JDemetra+ to carry out calculations

For the purpose of aggregating sub-indices or factoring sub-indices out of the overall index, as well as for calculating growth contributions for chained indices, use can be made of production line of the chain link programmes KIX, which are accessible through JDemetra+ as supplementary modules. When calculating the quarterly chained IIE, the Bundesbank uses the plug-in KIX CC (continuous chaining) that has been specifically designed for this purpose. The relevant software is available to the general public at https://github.com/bbkrd/KIX.

10 See Ribe (1999).

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